Free Fermion Systems: Topological Classification and Real-Space Invariants

Abstract: One of the major progress of modern condensed matter physics is the discovery of topological phases beyond Landau's paradigm—phases that are characterized by topology besides symmetries. In this thesis, we address topological phases of free fermionic systems by considering their topological classification and real-space invariants.

Although the theory for topological classification is fairly complete in momentum space, essentially based on the topological classification of fiber bundles, the theory in real space is more difficult. In this thesis, we discuss a formula for the Z_2 invariant of topological insulators. As a real-space formula, it is valid with or without translational invariance. Moreover, our formula is a local expression, in the sense that the contributions mainly come from quantities near a point. It is the local nature of this invariant that guarantees the existence of gapless mode on the boundary. Based on almost commute matrices, we provide a method to approximate this invariant with local information. The validity of the formula and the approximation method is rigorously proved.

The topological classification problem can be extended to non-Hermitian systems, an effective theory for systems with loss and gain. In this thesis, we propose a novel framework towards the topological classification of non-Hermitian band structures. Different from previous K-theoretical approaches, this approach is homotopical, which enables us to find more topological invariants. We find that the whole classification set is decomposed into several sectors, based on the braiding of energy levels. Each sector can be further classified based on the topology of eigenstates (wave functions). Due to the interplay between energy level braiding and eigenstates topology, we find some torsion invariants, which only appear in the non-Hermitian world. We further prove that these new topological invariants are unstable, in the sense that adding more bands will trivialize these invariants.

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